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# Holographic supergravity dual to three dimensional $\mathcal{N} = 2$ gauge theory

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Abstract: By examining the previously known holographic $\mathcal{N} = 2$ supersymmetric renormalization group flow solution in four dimensions, we describe the mass-deformed Bagger-Lambert theory, that has $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$ symmetry, by the addition of mass term for one of the four adjoint chiral superfields as its dual theory. A further detailed correspondence between fields of $AdS_4$ supergravity and composite operators of the infrared field theory is obtained.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence.

Contents



## 1. Introduction

The holographic theory on M2-branes is given by an $\mathcal{N} = 8$ supersymmetric theory with eight scalars, eight fermions and sixteen supercharges. The $AdS_4 \times \mathbf{S}^7$ background yields the holographic dual of a strongly coupled superconformal fixed point [1, 2]. By lifting the renormalization group(RG) flow in four dimensions [3, 4] to eleven dimensions, the M-theory solutions [5] which are holographic duals of flows of the maximally supersymmetric theory in three dimensions are examined. Giving one of four complex superfields a mass leads to an $\mathcal{N} = 2$ supersymmetric flow(four supersymmetries) to a new superconformal fixed point. The vacuum expectation values of remaining three complex superfields parametrize the Coulomb branch at this fixed point. A M2-brane probe analysis of the supergravity solution shows a three complex-dimensional space of moduli for the brane probe [6]. However, the microscopic configuration of coincident M2-branes was still lacking.

Recently, Bagger and Lambert(BL) have proposed a Lagrangian to describe the low energy dynamics of coincident M2-branes in [7]. See also relevant papers [8 − 11]. This BL theory is three dimensional $\mathcal{N} = 8$ supersymmetric field theory with SO(8) global symmetry based on new algebraic structure, 3-algebra. In particular, 3-algebra with Lorentzian signature was proposed by [12 − 14]. The generators of the 3-algebra are the generators of an arbitrary semisimple Lie algebra plus two additional null generators. This theory with gauge group SU($N$) is a good candidate for the theory of $N$ coincident M2-branes. We list some relevant works on the BL theory, from various different point of views, in [15]–[47].

In this paper, starting from the first order differential equations, that are the supersymmetric flow solution in four dimensional $\mathcal{N} = 8$ gauged supergravity interpolating between an exterior $AdS_4$ region with maximal supersymmetry and an interior $AdS_4$ with one quarter of the maximal supersymmetry, we want to interpret this as the RG flow in $\mathcal{N} = 8$

BL theory which has OSp(8|4) symmetry broken to an $\mathcal{N}=2$ theory which has OSp(2|4) symmetry by the addition of a mass term for one of the four adjoint chiral superfields. A precise correspondence is obtained between fields of bulk supergravity in the $AdS_4$ region and composite operators of the IR field theory in three dimensions.[1] Since the Lagrangian is known, one can check how the supersymmetry breaks for specific deformation and can extract the correct full superpotential including the superpotential before the deformation also. One would like to see the three dimensional analog of Leigh-Strassler [49] RG flow in mass-deformed BL theory in three dimensions by looking at its holographic dual theory in four dimensions along the line of [5].

In section 2, we review the supergravity solution in four dimensions in the context of RG flow, describe two supergravity critical points and present the supergravity multiplet in terms of $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$ invariant ones.[2]

In section 3, we deform BL theory by adding one of the mass term among four chiral superfields, along the lines of [51, 52], write down the superpotential in $\mathcal{N}=2$ superfields and describe the scale dimensions for the superfields at UV and IR.

In section 4, the OSp(2|4) representations(energy, spin, hypercharge) and $\mathrm{SU}(3)_I$ representations in the supergravity mass spectrum for each multiplet at the $\mathcal{N}=2$ critical point and the corresponding $\mathcal{N}=2$ superfield in the boundary gauge theory are given. The Kahler potential at IR is obtained.

In section 5, we end up with the future directions.

## 2. The holographic $\mathcal{N}=2$ supersymmetric RG flow in four dimensions

By gauging the SO(8) subgroup of $E_7$ in the global $E_7$ × local SU(8) supergravity [53], de Wit and Nicolai [54] constructed a four-dimensional supergravity theory. This theory has self-interaction of a single massless $\mathcal{N}=8$ supermultiplet of spins $(2, \frac{3}{2}, 1, \frac{1}{2}, 0^{\pm})$ but with local SO(8) × local SU(8) invariance. It is well known [55] that the 70 real scalars of $\mathcal{N}=8$ supergravity live on the coset space $E_{7(7)}/\mathrm{SU}(8)$ because 63 fields may be gauged away by an SU(8) rotation and are described by an element of the fundamental 56-dimensional representation of $E_7$. Then the effective nontrivial potential arising from SO(8) gauging can be written in compact form. The complex self-dual tensor describes the 35 scalars and 35 pseudo-scalar fields of $\mathcal{N}=8$ supergravity. After gauge fixing, one does not distinguish between SO(8) and SU(8) indices. The full supersymmetric solution where both scalars and pseudo-scalars vanish yields SO(8) vacuum state with $\mathcal{N}=8$ supersymmetry. Note that SU(8) is not a symmetry of the vacuum.

It is known that, in $\mathcal{N}=8$ supergravity, there also exists a $\mathcal{N}=2$ supersymmetric, $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$ invariant vacuum [56]. To reach this critical point, one has to turn

[1]For the $AdS_5 \times \mathbf{S}^5$ background with D3-branes, it is well known in [48] that the holographic dual is studied for the flow of $\mathcal{N}=4$ super Yang-Mills theory to the $\mathcal{N}=1$ supersymmetric Leigh-Strassler fixed point [49]. There exist earlier works on the M-theory flow solutions in 11 dimensions [50].

[2]We put the index $I$ in SU(3) group for "invariance" in order to emphasize that along the flow SU(3) group is preserved. The index $Y$ in $\mathrm{U}(1)_Y$ is for the hypercharge in the context of $AdS_4$ supergravity and is related to $\mathrm{U}(1)_R$ charge in the context of boundary gauge theory.



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| Symmetry | $\lambda$ | $\lambda'$ | V | W |
|---|---|---|---|---|
| SO(8) | 0 | 0 | $-6g^2$ | 1 |
| $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$ | $\sqrt{2}\sinh^{-1}\frac{1}{\sqrt{3}}$ | $\sqrt{2}\sinh^{-1}\frac{1}{\sqrt{2}}$ | $-\frac{9\sqrt{3}}{2}g^2$ | $\frac{3^{\frac{3}{4}}}{2}$ |

**Table 1:** Summary of two critical points with symmetry group, supergravity fields, scalar potential and superpotential.

on expectation values of both scalar $\lambda$ and pseudo-scalar $\lambda'$ fields where the completely antisymmetric self-dual and anti-self-dual tensors are invariant under $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$. Therefore 56-beins can be written as $56 \times 56$ matrix whose elements are some functions of scalar and pseudo-scalars. Then the $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$-invariant scalar potential of $\mathcal{N} = 8$ supergravity is given by [57, 56, 3]

$$V(\lambda, \lambda') = g^2 \left[ \frac{16}{3} \left( \frac{\partial W}{\partial \lambda} \right)^2 + 4 \left( \frac{\partial W}{\partial \lambda'} \right)^2 - 6W^2 \right] \tag{2.1}$$

where $g$ is SO(8) gauge coupling constant and the superpotential can be written as [3, 5]

$$W(\lambda, \lambda') = \frac{1}{16} e^{-\frac{1}{2\sqrt{2}}\lambda - \sqrt{2}\lambda'} \left( 3 - e^{\sqrt{2}\lambda} + 6e^{\sqrt{2}\lambda'} + 3e^{2\sqrt{2}\lambda'} + 6e^{\sqrt{2}(\lambda+\lambda')} - e^{\sqrt{2}(\lambda+2\lambda')} \right) \tag{2.2}$$

There are two critical points and we summarize these in table 1.

- SO(8) critical point

  There is well-known, trivial critical point at which all the scalars vanish($\lambda = \lambda' = 0$) and whose cosmological constant $\Lambda = -6g^2$ from (2.1) and which preserves $\mathcal{N} = 8$ supersymmetry.

- $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$ critical point

  There is a critical point at $\lambda = \sqrt{2}\sinh^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and $\lambda' = \sqrt{2}\sinh^{-1}\left(\frac{1}{\sqrt{2}}\right)$ and the cosmological constant $\Lambda = -\frac{9\sqrt{3}}{2}g^2$. This critical point has an unbroken $\mathcal{N} = 2$ supersymmetry.

For the supergravity description of the nonconformal RG flow from one scale to another connecting the two critical points, the three dimensional Poincare invariant metric takes the form $ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2$ where $\eta_{\mu\nu} = (-,+,+)$ and $r$ is the coordinate transverse to the domain wall. Then the supersymmetric flow equations [3, 5] with (2.2) are described as

$$\frac{d\lambda}{dr} = \frac{8}{3}\sqrt{2}g\frac{\partial W}{\partial \lambda}, \qquad \frac{d\lambda'}{dr} = 2\sqrt{2}g\frac{\partial W}{\partial \lambda'}, \qquad \frac{dA}{dr} = -\sqrt{2}gW. \tag{2.3}$$

The $AdS_4$ geometries at the end points imply conformal symmetry in the UV and IR limits of the field theory. We'll return to this when we discuss about the Kahler potential in section 4.



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Since the unbroken group symmetry at the stationary point is $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$, the fields of the $\mathcal{N}=8$ theory, transforming in SO(8) representations, should be decomposed into $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$ representations. From the quadratic fermion terms of the gauged $\mathcal{N}=8$ supergravity Lagrangian [56], there exist the massless and massive graviton mass terms. According to the decomposition $\mathrm{SO}(8) \to \mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$, the spin $\frac{3}{2}$ field breaks into [56]

$$\mathbf{8} \to \left[\mathbf{1}_{\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2}}\right] \oplus \mathbf{3}_{\frac{1}{6}} \oplus \overline{\mathbf{3}}_{-\frac{1}{6}}, \tag{2.4}$$

and the two singlets in square bracket correspond to the massless graviton of the $\mathcal{N}=2$ theory. The other terms in the quadratic fermion terms of the gauged $\mathcal{N}=8$ supergravity Lagrangian provide the spin $\frac{1}{2}$ masses and contain the Goldstino mass term. It turns out that there is no octet term and so the octet mass term vanishes. The tensors in gauged $\mathcal{N}=8$ supergravity [54] have SU(8) indices where the upper index transforms in $\mathbf{8}$ and the lower index transforms in $\overline{\mathbf{8}}$. Using the charge normalization of [56], one assigns the charges of $\frac{1}{6}$ and $-\frac{1}{6}$ to the lower indices $a$ and $\bar{a}$ respectively where $a=1,2,3$ and the lower indices 4 and $\bar{4}$ should be assigned charges $\frac{1}{2}$ and $-\frac{1}{2}$ respectively. These charges appear in (2.4). A new complex basis is introduced with an index $A$ and $\bar{A}$ and the $\mathbf{8}$ of SO(8) in cartesian system is relabelled by $A$ and $\bar{A}$ where $A=a,4$ and $\bar{A}=\bar{a},\bar{4}$.

From the branching rule of SO(8) into $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$, the spin $\frac{1}{2}$ field transforms as [56]

$$\begin{aligned} \mathbf{56} \to\ & \mathbf{1}_{\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2}} \oplus \mathbf{6}_{-\frac{1}{6}} \oplus \overline{\mathbf{6}}_{\frac{1}{6}} \oplus \mathbf{1}_{-\frac{1}{2}} \oplus \mathbf{1}_{\frac{1}{2}} \oplus \left[\mathbf{8}_{\frac{1}{2}} \oplus \mathbf{8}_{-\frac{1}{2}}\right] \\ & \oplus\ \mathbf{3}_{\frac{1}{6}} \oplus \mathbf{3}_{-\frac{5}{6}} \oplus \mathbf{3}_{\frac{1}{6}} \oplus \overline{\mathbf{3}}_{-\frac{1}{6}} \oplus \overline{\mathbf{3}}_{\frac{5}{6}} \oplus \overline{\mathbf{3}}_{-\frac{1}{6}} \oplus \left[\mathbf{3}_{\frac{1}{6}} \oplus \overline{\mathbf{3}}_{-\frac{1}{6}}\right], \end{aligned} \tag{2.5}$$

and the six Goldstino modes that are absorbed into massive spin $\frac{3}{2}$ fields are identified with triplets and anti-triplets in square bracket and the two octets in square bracket correspond to the massless vector multiplets of the $\mathcal{N}=2$ theory. The decomposition of the vector fields with respect to SO(8) [56]

$$\mathbf{28} \to \mathbf{1}_0 \oplus \mathbf{3}_{\frac{2}{3}} \oplus \mathbf{3}_{-\frac{1}{3}} \oplus \mathbf{3}_{-\frac{1}{3}} \oplus \overline{\mathbf{3}}_{-\frac{2}{3}} \oplus \overline{\mathbf{3}}_{\frac{1}{3}} \oplus \overline{\mathbf{3}}_{\frac{1}{3}} \oplus [\mathbf{1}_0] \oplus [\mathbf{8}_0] \tag{2.6}$$

implies that the singlet in square bracket corresponds to the massless graviton of the $\mathcal{N}=2$ theory while the octet in square bracket corresponds to the massless vector multiplets of the $\mathcal{N}=2$ theory. Finally from the branching rule for spin 0 field [56]

$$\begin{aligned} \mathbf{70} \to\ & \mathbf{1}_0 \oplus \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \mathbf{1}_0 \oplus \mathbf{1}_{-1} \oplus [\mathbf{8}_0 \oplus \mathbf{8}_0] \oplus \mathbf{3}_{-\frac{1}{3}} \oplus \overline{\mathbf{3}}_{\frac{1}{3}} \oplus \mathbf{6}_{\frac{1}{3}} \oplus \mathbf{6}_{-\frac{2}{3}} \oplus \overline{\mathbf{6}}_{-\frac{1}{3}} \oplus \overline{\mathbf{6}}_{\frac{2}{3}} \\ & \oplus \left[\mathbf{1}_0 \oplus \mathbf{3}_{\frac{2}{3}} \oplus \mathbf{3}_{-\frac{1}{3}} \oplus \mathbf{3}_{-\frac{1}{3}} \oplus \overline{\mathbf{3}}_{-\frac{2}{3}} \oplus \overline{\mathbf{3}}_{\frac{1}{3}} \oplus \overline{\mathbf{3}}_{\frac{1}{3}}\right], \end{aligned} \tag{2.7}$$

the two octets in square bracket correspond to the massless vector multiplets of the $\mathcal{N}=2$ theory and the nineteen Goldstone bosons modes are identified with singlet, triplets and anti-triplets in square bracket. Their quantum numbers are in agreement with those of massive vectors in (2.6).

Finally, spin 2 field has the breaking $\mathbf{1} \to \mathbf{1}_0$ and is located at $\mathcal{N}=2$ massless graviton multiplet.



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We'll rearrange (2.4), (2.5), (2.6) and (2.7) in the context of supergravity multiplet with corresponding OSp(2|4) quantum numbers in section 4. The singlets are placed at long massive vector multiplet, triplets and anti-triplets are located at short massive gravitino multiplet and sextets and anti-sextets sit in short massive hypermultiplet.

## 3. An $\mathcal{N} = 2$ supersymmetric membrane flow in three dimensional deformed BL theory

The original BL Lagrangian [7] consists of the Chern-Simons terms, the kinetic terms for matter fields, the Yukawa term and the potential term with supersymmetry transformations on the gauge and matter fields. The BF Lorentzian Lagrangian [12 − 14] can be obtained by choosing structure constant of BL theory appropriately with a Lorentzian bi-invariant metric. Then the Chern-Simons terms of BL theory become BF term and the kinetic terms for matter fields contain B-dependent terms besides other derivative terms. The simplest mass deformation to the BL Lagrangian is to add the single fermion mass term with modified supersymmetry transformations and other terms in the Lagrangian due to this deformation term [52, 51]. The aim of the first part in this section is to introduce the several mass terms for the fermion in the original BL Lagrangian. This procedure should preserve the exact $\mathcal{N} = 2$ supersymmetry. We determine what is the correct expression for the bosonic mass terms in the modified Lagrangian we should add to the original BL Lagrangian.

Let us consider the deformed BL theory by adding four mass parameters $m_1, m_2, m_3$ and $m_4$ to the BL theory Lagrangian, compared to [51] where there are three mass parameters.[3] See also the relevant paper by [52] on the mass deformation. Then the fermionic mass terms[4] from [7] are given by

$$\mathcal{L}_{f.m.} = -\frac{i}{2} h_{ab} \bar{\Psi}^a \left( m_1 \Gamma^{3579} + m_2 \Gamma^{35810} + m_3 \Gamma^{36710} - m_4 \Gamma^{3689} \right) \Psi^b. \tag{3.1}$$

Here the indices $a, b, \ldots$ run over the adjoint of the Lie algebra for BL theory(and those indices run over the adjoint plus $+, -$ for BF Lorentzian model). Then the corresponding fermionic supersymmetric transformation gets modified by

$$\delta_m \Psi^a = \left( m_1 \Gamma^{3579} + m_2 \Gamma^{35810} + m_3 \Gamma^{36710} - m_4 \Gamma^{3689} \right) X_I^a \Gamma_I \epsilon. \tag{3.2}$$

We impose three constraints on the $\epsilon$ parameter that satisfies the $\frac{1}{4}$ BPS condition(the number of supersymmetries is four) $\Gamma^{5678}\epsilon = \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = -\epsilon$.[5] Now we introduce

[3] This paragraph is based on the discussion with K. Hosomichi intensively.

[4] The self-dual and anti self-dual tensors that are invariant under the $SU(3)_I \times U(1)_Y$ in $\mathcal{N} = 8$ gauged supergravity are given by $X^+_{ijkl} = +[(\delta^{1234}_{ijkl} + \delta^{5678}_{ijkl}) + (\delta^{1256}_{ijkl} + \delta^{3478}_{ijkl}) + (\delta^{1278}_{ijkl} + \delta^{3456}_{ijkl})]$ and $X^-_{ijkl} = -[(\delta^{1357}_{ijkl} - \delta^{2468}_{ijkl}) + (\delta^{1368}_{ijkl} - \delta^{2457}_{ijkl}) + (\delta^{1458}_{ijkl} - \delta^{2367}_{ijkl}) - (\delta^{1467}_{ijkl} - \delta^{2358}_{ijkl})]$. The choice of [51] for the mass parameters corresponds to the self-dual tensor for the indices $1234, 1256$, and $1278$ while the choice of this paper for the mass parameters corresponds to the anti self-dual tensor for the indices $1357, 1368, 1458$ and $1467$ if we shift all the indices by adding 2. For example, the indices 3689 in (3.1) play the role of 1467 in above anti self-dual tensor.

[5] These indices $5678, 56910$ and $78910$ can be interpreted as $3456, 3478$ and $5678$ in $X^+_{ijkl}$ of footnote 4 respectively.

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the bosonic mass term which preserves $\mathcal{N}=2$ supersymmetry and determine $(m^2)_{IJ}$:

$$\mathcal{L}_{b.m.} = -\frac{1}{2} h_{ab} X^a_I (m^2)_{IJ} X^b_J. \tag{3.3}$$

Using the supersymmetry variation for $X^a_I$, $\delta X^a_I = i\bar{\epsilon}\Gamma_I \Psi^a$, and the supersymmetry variation for $\Psi^a$ by the equation (3.2), the variation for the bosonic mass term (3.3) plus the fermionic mass term (3.1) leads to

$$\delta\mathcal{L} = i h_{ab} X^a_I (m^2)_{IJ} \bar{\Psi}^b \Gamma_J \epsilon - i h_{ab} \bar{\Psi}^a \left(m_1 \Gamma^{3579} + m_2 \Gamma^{35810} + m_3 \Gamma^{36710} - m_4 \Gamma^{3689}\right)^2 X^b_I \Gamma_I \epsilon. \tag{3.4}$$

In order to vanish this, the bosonic mass term $(m^2)_{IJ}\Gamma_J$, by computing the mass term for second term of (3.4) explicitly,[6] should take the form

$$\begin{aligned} &(m_1 - m_2 - m_3 + m_4)^2 (\Gamma_3 + \Gamma_4) + (m_1 - m_2 + m_3 - m_4)^2 (\Gamma_5 + \Gamma_6) \\ &+ (m_1 + m_2 - m_3 - m_4)^2 (\Gamma_7 + \Gamma_8) + (m_1 + m_2 + m_3 + m_4)^2 (\Gamma_9 + \Gamma_{10}). \end{aligned} \tag{3.5}$$

In particular, when all the mass parameters are equal $m_1 = m_2 = m_3 = m_4 \equiv m$, then the diagonal bosonic mass term in (3.5) has nonzero components only for 99 and 1010 and other components$(33, 44, 55, 66, 77$ and $88)$ are vanishing:[7]

$$(m^2)_{IJ} = \mathrm{diag}(0, 0, 0, 0, 0, 0, 16m^2, 16m^2). \tag{3.6}$$

Of course, the quartic terms for $X^a_I$ to the Lagrangian for our mass deformation can be fixed similarly, as in [51]. Let us introduce the four complex $\mathcal{N}=2$ superfields as follows:

$$\begin{aligned} \Phi_1 &= X_3 + iX_4 + \cdots, \qquad & \Phi_2 &= X_5 + iX_6 + \cdots, \\ \Phi_3 &= X_7 + iX_8 + \cdots, \qquad & \Phi_4 &= X_9 + iX_{10} + \cdots \end{aligned} \tag{3.7}$$

where we do not include the $\mathcal{N}=2$ fermionic fields. Or one can introduce these chiral superfields with an explicit $\mathrm{SU}(4)_I$ fundamental representation as follows:

$$\Phi_A, \qquad A = (a, 4), \qquad a = 1, 2, 3.$$

Recall from section 2 that the **8** of SO(8) is relabelled by $A$ and $\bar{A}$ where $A = a, 4$ and $\bar{A} = \bar{a}, \bar{4}$. Then the subset $\Phi_a$ where $a = 1, 2, 3$ constitute a **3** representation of SU(3) inside SU(4). The potential in the BL theory [7] is given by

$$\frac{1}{3\kappa^2} h_{ab} f_{cde}{}^a X^c_I X^d_J X^e_K f_{fgh}{}^b X^f_I X^g_J X^h_K$$

where $\kappa$ is a Chern-Simons coefficient. In terms of $\mathcal{N}=2$ superfields, this contains the following expressions

$$\frac{2}{\kappa^2} h_{ab} f_{cde}{}^a f_{fgh}{}^b \left[\Phi^c_1 \Phi^d_2 \Phi^e_3 \bar{\Phi}^f_1 \bar{\Phi}^g_2 \bar{\Phi}^h_3 + \text{three other terms}\right]$$

[6]The relevant terms become $m_1^2 + m_2^2 + m_3^2 + m_4^2 - 2(m_1 m_4 + m_2 m_3)\Gamma^{5678} + 2(m_1 m_2 + m_3 m_4)\Gamma^{78910} + 2(m_1 m_3 + m_2 m_4)\Gamma^{56910}$ explicitly.

[7]This resembles the structure of $A_1^{IJ}$ tensor of $AdS_4$ supergravity where the $A_1^{IJ}$ tensor has two distinct eigenvalues with degeneracies 6 and 2 respectively. The degeneracy 2 is quite related to the $\mathcal{N}=2$ supersymmetry. For the maximal supersymmetric case [51], all the diagonal mass matrix elements are equal and nonzero and this reflects the fact that the $A_1^{IJ}$ tensor has eight equal eigenvalues with degeneracies 8.

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by using the relation (3.7) between the component fields and superfields and a fundamental identity is used. This provides the superpotential: $\frac{\sqrt{2}}{\kappa} f_{abcd} f^{ABCD}\, \mathrm{Tr}\, \Phi_A^a \Phi_B^b \Phi_C^c \Phi_D^d$. Then this superpotential possesses $\mathrm{SU}(4)_I$ global symmetry.[8]

In $\mathcal{N}=2$ language, the superpotential consisting of the mass term (3.6) and quartic term, where we redefine $\Phi_4$ by diagonalizing the mass matrix and introducing the new bosonic variables $X_9^a$ and $X_{10}^a$, is given by

$$W = \frac{1}{2} M h_{ab}\, \mathrm{Tr}\, \Phi_4^a \Phi_4^b + \frac{\sqrt{2}}{\kappa} f_{abcd} f^{ABCD}\, \mathrm{Tr}\, \Phi_A^a \Phi_B^b \Phi_C^c \Phi_D^d. \tag{3.8}$$

The global symmetry $\mathrm{SU}(4)_I$ of SO(8) is broken to $\mathrm{SU}(3)_I$. The second term is the superpotential required by $\mathcal{N}=8$ supersymmetry as we mentioned above and the first term breaks $\mathcal{N}=8$ down to $\mathcal{N}=2$. The theory has matter multiplet in three flavors $\Phi_1, \Phi_2$ and $\Phi_3$ transforming in the adjoint. The $\mathrm{SO}(8)_R$ symmetry of the $\mathcal{N}=8$ gauge theory is broken to $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$ where the former is a flavor symmetry under which the matter multiplet forms a triplet and the latter is the R-symmetry of the $\mathcal{N}=2$ theory. Therefore, we turn on the mass perturbation in the UV and flow to the IR. This maps to turning on certain scalar fields in the $AdS_4$ supergravity where the scalars approach to zero in the UV$(r \to \infty)$ and develop a nontrivial profile as a function of $r$ becoming more significantly different from zero as one goes to the IR$(r \to -\infty)$. We can integrate out the massive scalar $\Phi_4$ with adjoint index at a low enough scale and this results in the 6-th order superpotential $\mathrm{Tr}(f_{abc} f^{ABCD} \Phi_A^a \Phi_B^b \Phi_C^c)^2$.

The scale dimensions of four chiral superfields $\Phi_i (i=1,2,3,4)$ are $\Delta_i = \frac{1}{2}$ at the UV. This is because the sum of $\Delta_i$ is equal to the canonical dimension of the superpotential which is $3-1=2$ [59]. By symmetry, one arrives at $\Delta_i = \frac{1}{2}$. The beta function from the mass term of $\Phi_4$ in (3.8) leads to $\beta_M = M(2\Delta_4 - 2)$ [6]. Or one can compute the anomalous mass dimension $\gamma_i$ explicitly as follows [59]:

$$\begin{aligned}
\beta_{1,1,1,1} &\sim 4 \times (3-2) - 2 \times (3-1) + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4, \\
\beta_{0,0,0,2} &\sim 2 \times (3-2) - 2 \times (3-1) + 2\gamma_4 = -2 + 2\gamma_4.
\end{aligned} \tag{3.9}$$

The $\mathcal{N}=2$ supersymmetric gauge theory in three dimensions has a holomorphic superpotential and non-perturbative renormalizations of the superpotential are restricted by holomorphy. The form of (3.9) is a consequence of the non-renormalization theorem for superpotential in $\mathcal{N}=2$ supersymmetry in three dimensions. Then the vanishing of these (3.9) leads to $\gamma_1 = \gamma_2 = \gamma_3 = -\frac{1}{3}$ and $\gamma_4 = 1$. This imposes one relation between

[8] Note that in [58] appeared in the same daily distribution of the arXiv, the $\mathcal{N}=2$ superspace formalism for BL theory with gauge group $\mathrm{SU}(2) \times \mathrm{SU}(2)$ was found and the superpotential has $\mathrm{SU}(4)_I \times \mathrm{U}(1)_R$ global symmetry. When the normalization constants in the $\mathcal{N}=2$ superspace Lagrangian hold some relation, the $R$-symmetry is enhanced to SO(8) and further requirement on these constants allows this $\mathcal{N}=2$ superspace Lagrangian to reduce to the one in component Lagrangian. For BF Lorentzian model, it is not known yet how to write down the Lagrangian in $\mathcal{N}=2$ superspace formalism. So it is not clear at this moment how one can proceed further on the direction of BF Lorentzian model. Furthermore, the mass deformed BL theory with two M2-branes is equivalent to the mass deformed $\mathrm{U}(2) \times \mathrm{U}(2)$ Chern-Simons gauge theory of [15] with level $k=1$ or $k=2$.



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$M$ and $\kappa$ suggesting that the theory has a fixed line of couplings. Furthermore, the conformal dimension for $\Phi_4$ is given by $\Delta_4 = \frac{1}{2}(1+\gamma_4) = 1$. This comes from the relation $M(-2+2\Delta_4) = \frac{M}{2}(-2+2\gamma_4)$ using the equation (1) of [59]. Similarly, $\Delta_1 = \Delta_2 = \Delta_3 = \frac{1}{3}$. In other words, the IR values of scaling dimensions are $\Delta_4 = 1$ and $\Delta_i = \frac{1}{3}(i = 1,2,3)$. Then the $\mathrm{U}(1)_R$ symmetry acts on $\Phi_1, \Phi_2, \Phi_3$ and $\Phi_4$ with charges $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1)$ which are correctly related to the above anomalous dimensions. So both terms in the superpotential (3.8) have R charge 2, as they must. If we allow the mass $M$ to rotate by a phase then we have a further $\mathrm{U}(1)$ symmetry under which $\Phi_i(i = 1,2,3)$ has charge $\frac{1}{3}$ and $\Phi_4$ has charge zero while the mass $M$ has charge 1.

In next section, the gauge invariant composites in the superconformal field theory at the IR are mapped to the corresponding supergravity bulk fields.

## 4. The OSp(2|4) spectrum and operator map between bulk and boundary theories

A further detailed correspondence between fields of $AdS_4$ supergravity in four dimensions and composite operators of the IR field theory in three dimensions is described in this section.

The even subalgebra of the superalgebra OSp(2|4) is a direct sum of subalgebras where $\mathrm{Sp}(4,R) \simeq \mathrm{SO}(3,2)$ is the isometry algebra of $AdS_4$ and the compact subalgebra SO(2) generates $\mathrm{U}(1)_R$ symmetry [60]. The maximally compact subalgebra is then $\mathrm{SO}(2)_E \times \mathrm{SO}(3)_S \times \mathrm{SO}(2)_Y$ where the generator of $\mathrm{SO}(2)_E$ is the hamiltonian of the system and its eigenvalues $E$ are the energy levels of states for the system, the group $\mathrm{SO}(3)_S$ is the roatation group and its representation $s$ describes the spin states of the system, and the eigenvalue $y$ of the generator of $\mathrm{SO}(2)_Y$ is the hypercharge of the state.

A supermultiplet, a unitary irreducible representations(UIR) of the superalgebra OSp(2|4), consists of a finite number of UIR of the even subalgebra and a particle state is characterized by a spin $s$, a mass $m$ and a hypercharge $y$. The relations between the mass and energy are given in [61] sometime ago.

Let us classify the supergravity multiplet which is invariant under $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$ and describe them in the three dimensional boundary theory.

- Long massive vector multiplet

  The conformal dimension $\Delta$, which is irrational and unprotected, is $\Delta = E_0$ and the $\mathrm{U}(1)_R$ charge is 0. The $\mathrm{U}(1)_R$ charge[9] is related to a hypercharge by

$$R = y. \tag{4.1}$$

  The $K(x, \theta^+, \theta^-)$ is a general "unconstrained" scalar superfield in the boundary theory. Since the Kahler potential evolves in the RG flow, the scalar field that measures the approach of the trajectory to the $\mathcal{N} = 2$ point sits in the supergravity multiplet

[9] The assignment of this $\mathrm{U}(1)_R$ charge is different from the one given in [62] where the SO(8) branching rule is the same as the present case because both theories have the same number of supersymmetries.

| Boundary Operator(BO) | Energy | Spin 0 | Spin $\frac{1}{2}$ | Spin 1 |
|---|---|---|---|---|
| $K = \left(\Phi_1\bar{\Phi}_1 + \Phi_2\bar{\Phi}_2 + \Phi_3\bar{\Phi}_3\right)^{\frac{3}{2}}$ | $E_0 = \frac{1}{2}(1+\sqrt{17})$ | $\mathbf{1}_0$ | | |
| | $E_0 + \frac{1}{2} = \frac{1}{2}(2+\sqrt{17})$ | | $\mathbf{1}_{\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2}}$ | |
| | $E_0 + 1 = \frac{1}{2}(3+\sqrt{17})$ | $\mathbf{1}_1 \oplus \mathbf{1}_0 \oplus \mathbf{1}_{-1}$ | | $\mathbf{1}_0$ |
| | $E_0 + \frac{3}{2} = \frac{1}{2}(4+\sqrt{17})$ | | $\mathbf{1}_{\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2}}$ | |
| | $E_0 + 2 = \frac{1}{2}(5+\sqrt{17})$ | $\mathbf{1}_0$ | | |

**Table 2:** The OSp(2|4) representations(energy, spin, hypercharge) and $SU(3)_I$ representations in the supergravity mass spectrum for long massive vector multiplet(corresponding to table 1 of [56]) at the $\mathcal{N}=2$ critical point and the corresponding $\mathcal{N}=2$ superfield in the boundary gauge theory.

dual to $K(x,\theta^+,\theta^-)$, as in $AdS_5$ supergravity [48]. This scalar field has a dimension $\frac{1}{2}(5+\sqrt{17})$ in the IR. We'll come back this issue at the end of this section. The corresponding OSp(2|4) representations and corresponding $\mathcal{N}=2$ superfield in three dimensions are listed in table 2. The relation between $\Delta$ and the mass for various fields can be found in [61]. For spin 0 and 1, their relations are given by $\Delta_\pm = \frac{3\pm\sqrt{1+\frac{m^2}{4}}}{2}$ where we have to choose the correct root among two cases as in [63] while for spin $\frac{1}{2}$, the explicit form is given by $\Delta = \frac{6+|m|}{4}$. Using these relations, one can read off the mass for each state.

- Short massive hypermultiplet

  The conformal dimension $\Delta$ is the $U(1)_R$ charge for the lowest component which can be written as $\Delta = \frac{E_0}{2} = |R|$. The $AdS_4$ supergravity multiplet corresponds to the chiral scalar superfield $\Phi_c(x,\theta^+)$ that satisfies $D^+_\alpha \Phi_c(x,\theta^+) = 0$ making the multiplet short [64].[10] That is, in the $\theta^+$ expansion, there are three component fields in the bulk. For the anti-chiral scalar superfield, one can see the similar structure. Since the massive field $\Phi_4$ is integrated out in the flow, the IR theory contains the massless chiral superfields $\Phi_1, \Phi_2, \Phi_3$ with $\Delta = \frac{1}{3}$ and $U(1)_R$ charge $\frac{1}{3}$ from the discussion of section 3 with (4.1). Then the bilinear of these chiral superfields by symmetrizing the two $SU(3)_I$ indices provides a symmetric representation of $SU(3)_I$, $\mathbf{6}$, corresponding to $\mathrm{Tr}\,\Phi_{(i}\Phi_{j)}$ and its conjugate representation $\bar{\mathbf{6}}$, corresponding to $\mathrm{Tr}\,\bar{\Phi}_{(i}\bar{\Phi}_{j)}$. Using the relations between the dimension and mass for spin 0 and $\frac{1}{2}$, one can also read off the mass for each state. The corresponding OSp(2|4) representations and corresponding superfield are listed in table 3.

- Short massive gravitino multiplet

  The conformal dimension $\Delta$ is the twice of $U(1)_R$ charge plus $\frac{3}{2}$ for the lowest component, $\Delta = E_0 = 2|R| + \frac{3}{2}$. This corresponds to spinorial superfield $\Phi_\alpha(x,\theta^+)$ that satisfies $D^{+\alpha}\Phi_\alpha(x,\theta^+) = 0$ [65]. Of course, this constraint makes the multiplet short.

[10] The conformal dimension $\Delta$ and $U(1)_R$ charge for $\theta^+_\alpha$ are $\frac{1}{2}$ and $\frac{1}{2}$ while the conformal dimension $\Delta$ and $U(1)_R$ charge for $\theta^-_\alpha$ are $\frac{1}{2}$ and $-\frac{1}{2}$.



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| Boundary Operator | Energy | Spin 0 | Spin $\frac{1}{2}$ |
|---|---|---|---|
| $\mathrm{Tr}\,\Phi_{(i}\Phi_{j)}$ | $E_0 = \frac{4}{3}$ | $\mathbf{6}_{-\frac{2}{3}} \oplus \bar{\mathbf{6}}_{\frac{2}{3}}$ | |
| complex | $E_0 + \frac{1}{2} = \frac{11}{6}$ | | $\mathbf{6}_{-\frac{1}{6}} \oplus \bar{\mathbf{6}}_{\frac{1}{6}}$ |
| | $E_0 + 1 = \frac{7}{3}$ | $\mathbf{6}_{\frac{1}{3}} \oplus \bar{\mathbf{6}}_{-\frac{1}{3}}$ | |

**Table 3:** The OSp(2|4) representations(energy, spin, hypercharge) and $\mathrm{SU}(3)_I$ representations in the supergravity mass spectrum for short massive hypermultiplet(corresponding to table 2 of [56]) at the $\mathcal{N}=2$ critical point and the corresponding $\mathcal{N}=2$ superfield in the boundary gauge theory where $E_0 = 2|y| = 2|R|$.

| B.O. | Energy | Spin 0 | Spin $\frac{1}{2}$ | Spin 1 | Spin $\frac{3}{2}$ |
|---|---|---|---|---|---|
| $\mathrm{Tr}\,W_\alpha \Phi_j$ | $E_0 = \frac{11}{6}$ | | $\mathbf{3}_{\frac{1}{6}} \oplus \bar{\mathbf{3}}_{-\frac{1}{6}}$ | | |
| complex | $E_0 + \frac{1}{2} = \frac{7}{3}$ | $\mathbf{3}_{-\frac{1}{3}} \oplus \bar{\mathbf{3}}_{\frac{1}{3}}$ | | $\mathbf{3}_{\frac{2}{3}} \oplus \mathbf{3}_{-\frac{1}{3}} \oplus \bar{\mathbf{3}}_{-\frac{2}{3}} \oplus \bar{\mathbf{3}}_{\frac{1}{3}}$ | |
| | $E_0 + 1 = \frac{17}{6}$ | | $\mathbf{3}_{\frac{1}{6}} \oplus \bar{\mathbf{3}}_{-\frac{1}{6}} \oplus \mathbf{3}_{-\frac{5}{6}} \oplus \bar{\mathbf{3}}_{\frac{5}{6}}$ | | $\mathbf{3}_{\frac{1}{6}} \oplus \bar{\mathbf{3}}_{-\frac{1}{6}}$ |
| | $E_0 + \frac{3}{2} = \frac{10}{3}$ | | | $\mathbf{3}_{-\frac{1}{3}} \oplus \bar{\mathbf{3}}_{\frac{1}{3}}$ | |

**Table 4:** The OSp(2|4) representations(energy, spin, hypercharge) and $\mathrm{SU}(3)_I$ representations in the supergravity mass spectrum for short massive gravitino multiplet(corresponding to table 3 of [56]) at the $\mathcal{N}=2$ critical point and the corresponding $\mathcal{N}=2$ superfield in the boundary gauge theory where $E_0 = 2|y| + \frac{3}{2} = 2|R| + \frac{3}{2}$.

In the $\theta^{\pm}$ expansion, the component fields in the bulk are located with appropriate quantum numbers. The massless chiral superfields $\Phi_1, \Phi_2, \Phi_3$ have $\Delta = \frac{1}{3}$ and $\mathrm{U}(1)_R$ charge $\frac{1}{3}$ as before. The gauge superfield $W_\alpha$ has $\Delta = \frac{3}{2}$ and $\mathrm{U}(1)_R$ charge $-\frac{1}{6}$ and its conjugate field has opposite $\mathrm{U}(1)_R$ charge $\frac{1}{6}$. Then one can identify $\mathrm{Tr}\,W_\alpha \Phi_j$ with $\mathbf{3}$ and $\mathrm{Tr}\,\bar{W}_\alpha \bar{\Phi}_j$ with $\bar{\mathbf{3}}$. The corresponding OSp(2|4) representations and corresponding superfield are listed in table 4. For spin $\frac{3}{2}$, the relation for the mass and dimension is given by $\Delta = \frac{6+|m+4|}{4}$ and for spin $0, 1$ and $\frac{1}{2}$, the previous relations hold.

- $\mathcal{N}=2$ massless graviton multiplet

  This can be identified with the stress energy tensor superfield $T^{\alpha\beta}(x, \theta^+, \theta^-)$ that satisfies the equations $D^+_\alpha T^{\alpha\beta} = 0 = D^-_\alpha T^{\alpha\beta}$ [64, 66]. In components, the $\theta^{\pm}$ expansion of this superfield has the stress energy tensor, the $\mathcal{N}=2$ supercurrents, and $\mathrm{U}(1)_R$ symmetry current. The conformal dimension $\Delta = 2$ and the $\mathrm{U}(1)_R$ charge is 0. This has protected dimension. The corresponding OSp(2|4) representations and corresponding superfield are listed in table 5. For spin 2, we have the relation $\Delta_{\pm} = \frac{3 \pm \sqrt{9 + \frac{m^2}{4}}}{2}$ and for massless case, this leads to $\Delta_+ = 3$.

- $\mathcal{N}=2$ massless vector multiplet

  This conserved vector current is given by a scalar superfield $J^A(x, \theta^+, \theta^-)$ that satisfies $D^{+\alpha} D^+_\alpha J^A = 0 = D^{-\alpha} D^-_\alpha J^A$ [64]. This transforms in the adjoint representation of $\mathrm{SU}(3)_I$ flavor group. The boundary object is given by $\mathrm{Tr}\,\bar{\Phi} T^A \Phi$ where the fla-

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| Boundary Operator | Energy | Spin 0 | Spin $\frac{1}{2}$ | Spin 1 | Spin $\frac{3}{2}$ | Spin 2 |
|---|---|---|---|---|---|---|
| $\mathrm{Tr}\,\bar{\Phi}T^A\Phi$ | $E_0 = 1$ | $\mathbf{8}_0$ | | | | |
| | $E_0 + \frac{1}{2} = \frac{3}{2}$ | | $\mathbf{8}_{\frac{1}{2}} \oplus \mathbf{8}_{-\frac{1}{2}}$ | | | |
| | $E_0 + 1 = 2$ | $\mathbf{8}_0$ | | $\mathbf{8}_0$ | | |
| $T^{\alpha\beta}$ | $E_0 = 2$ | | | $\mathbf{1}_0$ | | |
| | $E_0 + \frac{1}{2} = \frac{5}{2}$ | | | | $\mathbf{1}_{\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2}}$ | |
| | $E_0 + 1 = 3$ | | | | | $\mathbf{1}_0$ |

**Table 5:** The OSp(2|4) representations(energy, spin, hypercharge) and $\mathrm{SU}(3)_I$ representations in the supergravity mass spectrum for "ultra" short multiplets at the $\mathcal{N} = 2$ critical point and the corresponding $\mathcal{N} = 2$ superfields in the boundary gauge theory.

vor indices in $\Phi_i$ and $\bar{\Phi}_i$ are contracted and the generator $T^A$ is $N \times N$ matrix with $A = 1, 2, \ldots, N^2-1$. The conformal dimension $\Delta = 1$ and the $\mathrm{U}(1)_R$ charge is 0. This has also protected dimension. By taking a tensor product between $\mathbf{3}$ and $\bar{\mathbf{3}}$, one gets this octet $\mathbf{8}$ of $\mathrm{SU}(3)_I$ representation. The corresponding OSp(2|4) representations and corresponding superfield are listed in table 5 also.

Let us describe the Kahler potential more detail we mentioned in the long vector multiplet. The Kahler potential is found in [6], by looking at the 11 dimensional flow equation [5], as

$$K = \frac{1}{4}\tau_{M2} L^2 e^A \left(\rho^2 + \frac{1}{\rho^6}\right), \qquad \frac{dq}{dr} = \frac{2}{L\rho^2} q \tag{4.2}$$

where $\rho \equiv e^{\frac{\lambda}{4\sqrt{2}}}$ and $\chi \equiv \frac{\lambda'}{\sqrt{2}}$. The corresponding Kahler metric is given by [6]

$$ds^2 = \frac{1}{4q^2}\left(q\frac{d}{dq}\right)^2 K dq^2 + \left(q\frac{d}{dq}\right) K d\hat{x}^I d\hat{x}^I + \left(q^2\frac{d^2}{dq^2}\right) K (\hat{x}^I J_{IJ} d\hat{x}^J)^2 \tag{4.3}$$

where the coordinate $q$ is defined as $q \equiv w^1\bar{w}^1 + w^2\bar{w}^2 + w^3\bar{w}^3$ and the three complex coordinates are given by $w^1 = \sqrt{q}\left(\hat{x}^1 + i\hat{x}^2\right), w^2 = \sqrt{q}\left(\hat{x}^3 + i\hat{x}^4\right)$ and $w^3 = \sqrt{q}\left(\hat{x}^5 + i\hat{x}^6\right)$ on $\mathbf{C}^3$ and the $\hat{x}$'s are coordinates on an $\mathbf{S}^5$ of unit radius. So we reparametrize $\mathbf{C}^3$ with coordinates $\hat{x}^1, \ldots, \hat{x}^6$ and $q$. Here $J$ is an antisymmetric matrix with $J_{12} = J_{34} = J_{56} = 1$. The $d\hat{x}^I d\hat{x}^I$ is a metric on a round $\mathbf{S}^5$ and $(\hat{x}^I J_{IJ} d\hat{x}^J)^2$ is the U(1) fiber in the description of $\mathbf{S}^5$. Note that there is a relation $\frac{dK}{dr} = \tau_{M_2} L e^A$ [6]. The moduli space is parametrized by the vacuum expectation values of the three massless scalars $\Phi_1, \Phi_2$ and $\Phi_3$ denoted as $w^1, w^2$ and $w^3$. The $w^i (i = 1, 2, 3)$ transform in the fundamental representation $\mathbf{3}$ of $\mathrm{SU}(3)_I$ while their complex conjugates $\bar{w}^i$ transform in the anti-fundamental representation $\bar{\mathbf{3}}$.

At the UV end of the flow which is just $AdS_4 \times \mathbf{S}^7$, $A(r) \sim \frac{2}{L} r$ from the solution (2.3) for $A(r)$ and $W = 1$ from table 1. Moreover, the radial coordinate on moduli space $\sqrt{q} \sim e^{\frac{r}{L}} \sim e^{\frac{A(r)}{2}}$ from (4.2) by substituting $\rho = 1$ from table 1. Therefore, the Kahler potential from (4.2) behaves as $K \sim e^{A(r)} \sim q$. This implies that $K = \Phi_1\bar{\Phi}_1 + \Phi_2\bar{\Phi}_2 + \Phi_3\bar{\Phi}_3$ at the UV in the boundary theory. Since the scaling dimensions for $\Phi_i (i = 1, 2, 3)$ and its



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conjugate fields are $\frac{1}{2}$, the scaling dimension of $K$ is equal to 1 which is correct because it should have scaling dimension 1 "classically" from $\int d^3x \partial_\varphi \partial_{\bar{\varphi}} K \partial_\mu \varphi \partial^\mu \bar{\varphi}$ where $\varphi$ are the massless scalars with some scaling dimensions.

At the IR end of the flow, $A(r) \sim \frac{3^{\frac{3}{4}}}{L} r$ with $g \equiv \frac{\sqrt{2}}{L}$ from the solution (2.3) for $A(r)$ and $W = \frac{3^{\frac{3}{4}}}{2}$ from table 1. Moreover, $\sqrt{q} \sim e^{\frac{3^{-\frac{1}{4}} r}{L}} \sim e^{\frac{A(r)}{3}}$ from (4.2) by substituting $\rho = 3^{\frac{1}{8}}$ from table 1. Therefore, the Kahler potential behaves as $K \sim e^{A(r)} \sim q^{\frac{3}{2}}$. Then $K$ becomes $K = (\Phi_1 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2 + \Phi_3 \bar{\Phi}_3)^{\frac{3}{2}}$ in the boundary theory. Obviously, from the tensor product between $\mathbf{3}$ and $\bar{\mathbf{3}}$ of $\mathrm{SU}(3)_I$ representation, one gets a singlet $\mathbf{1}_0$ with $\mathrm{U}(1)_R$ charge 0. Note that $\Phi_i (i = 1, 2, 3)$ has $\mathrm{U}(1)_R$ charge $\frac{1}{3}$ while $\bar{\Phi}_i (i = 1, 2, 3)$ has $\mathrm{U}(1)_R$ charge $-\frac{1}{3}$. Since the scaling dimensions for $\Phi_i (i = 1, 2, 3)$ and its conjugate fields are $\frac{1}{3}$, the scaling dimension of $K$ is 1 which is consistent with "classical" value as before. The corresponding Kahler metric (4.3) provides the Kahler term in the action. For the superfield $K(x, \theta^+, \theta^-)$, the action looks like $\int d^3x d^2\theta^+ d^2\theta^- K(x, \theta^+, \theta^-)$. The component content of this action can be worked out straightforwardly using the projection technique. This implies that the highest component field in $\theta^\pm$-expansion, the last element in table 2, has a conformal dimension $\frac{1}{2}(5 + \sqrt{17})$ in the IR as before.[11]

We have presented the gauge invariant combinations of the massless superfields of the gauge theory whose scaling dimensions and $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$ quantum numbers exactly match the four short multiplets in tables 3, 4, 5 observed in the supergravity. There exists one additional long multiplet in table 2 which completes the picture.

## 5. Conclusions and outlook

By studying the mass-deformed Bagger-Lambert theory, preserving $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$ symmetry, with the addition of mass term for one of the four adjoint chiral superfields, one identifies an $\mathcal{N} = 2$ supersymmetric membrane flow in three dimensional deformed BL theory with the holographic $\mathcal{N} = 2$ supersymmetric RG flow in four dimensions. Therefore, the $\mathcal{N} = 8$ gauged supergravity critical point is indeed the holographic dual of the mass-deformed $\mathcal{N} = 8$ BL theory. So far, we have focused on the particular mass deformation (3.1) preserving $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$ symmetry. It would be interesting to discover

[11] So far we have considered the leading behavior of Kahler potential at the two end points of UV and IR. This can be understood from the "classical" description of scaling dimension above also. However, one can look at next-to-leading order "quantum" corrections to this Kahler potential. The exact expression for the Kahler potential along "the whole flow" is given by (4.2). One can easily obtain the asymptotic behaviors of $A(r)$ and $\rho(r)$ around IR region. The former can be determined through the last one of first order differential equations (2.3) by expanding the superpotential $W$ around $\rho = 3^{\frac{1}{8}}$ and $\chi = \frac{1}{2} \cosh^{-1} 2$ while the latter can be obtained through the first equation of (2.3) by expanding the right hand side of that equation around IR fixed point values $\rho = 3^{\frac{1}{8}}$ and $\chi = \frac{1}{2} \cosh^{-1} 2$. Then one expects that the irrational piece $3 - \sqrt{17}$ from the mass spectrum found in [3] arises in the exponent of next-to-leading order $r$-dependent term in $\rho$ and $\chi$. The coefficient appearing in the next-to-leading order of Kahler potential is related to the mass of $\Phi_4$ via M2-brane probe analysis. One can approximate the Kahler potential by $K \sim (\Phi_1 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2 + \Phi_3 \bar{\Phi}_3)^{\frac{3}{2}}$ up to leading order at "the IR fixed point" but along the flow around the IR, in general, the Kahler potential is given by (4.2). For the relevant work on $AdS_5 \times \mathbf{S}^5$ compactification with D3-branes, see the section 2.5 of [6] for example.

all the possible classification for the mass deformations and see how they appear in the $AdS_4 \times \mathbf{S}^7$ background where some of them are nonsupersymmetric and some of them are supersymmetric [67].

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